IAIS Base Yield Curve Methodology for ICS Version 2.0

1. Background

The methodology used to discount insurance liabilities is a key element in the valuation of current estimates and has a significant overall impact in the final outcome of the ICS calculations. For this reason, it has been at the centre of IAIS work on valuation over the last few years.

In the context of the Market-Adjusted Valuation (MAV), the main objective of providing IAIS specified discount curves is comparability. In line with the ICS Principles, it is of paramount importance that the valuation methodology provides an appropriate balance between risk-sensitivity and stability, as well as a consistent approach between assets and liabilities. The discounting methodology plays a crucial role in the achievement of these objectives.

Since the start of Field Testing work by the IAIS, the approach taken for discounting is to prescribe yield curves for the 35 most traded currencies and provide the methodology for determining those yield curves for Volunteer IAIGs that operate in other markets that are not covered by the prescribed yield curves. The IAIS published basis risk-free yield curves for integer maturities from 1 to 89 years.

The prescribed yield curves by currency were created by:

- determining base yield curves; and,

- applying an adjustment to that base yield curve.

The aim of this paper is to provide a comprehensive overview of the approach taken so far by the IAIS on the different technical elements which constitute the base yield curve methodology. The paper does not address the topic of the adjustments to the base yield curve.

# 2. Three Segment Approach

2.1. Previous Approach

When the IAIS first published base yield curves, for the purpose of the 2014 Field Testing exercise, a 2 segment approach was adopted for simplicity. A first segment, based on market information (government bonds or swaps), was defined up to 30 years for all currencies and, after that point, the curve was simply considered to be a flat line (i.e. the "flat after 30 years" assumption).

In response to feedback received through Field Testing and the 2014 ICS Consultation, the approach to the construction of the base yield curve was refined.

The most significant change was the abandonment of the flattening of the yield curve after the 30-year point. From 2015 onwards, the design of the base yield curve evolved into a three-segment design:

- Segment 1: Liquid segment based on market information;

- Segment 2: Extrapolation/gradation between first and third segments; and

- Segment 3 (convergence): Long Term Forward Rate (LTFR) that begins at year 60 for all currencies. The LTFR was determined using a macroeconomic approach based on information from the Organisation for Economic Co-operation and Development (OECD).

Figure 1: Three segment base yield curve design
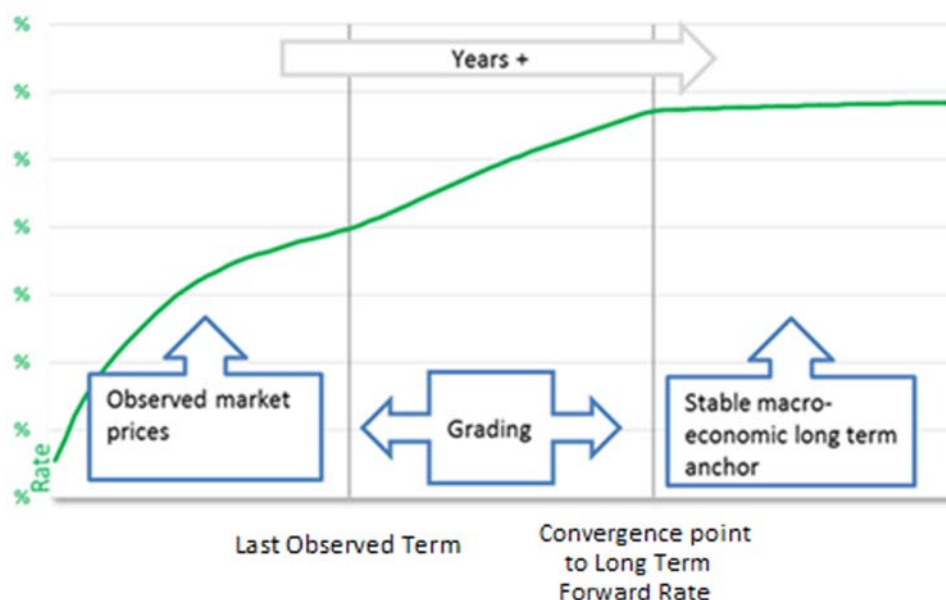
This revised approach was welcomed by Volunteer IAIGs and stakeholders, as it allowed for a balance between the reflection of jurisdiction specific market conditions (e.g. by basing Segment 1 on market information and setting a jurisdiction-specific Last Observed Term and Long Term Forward Rate) and stability (e.g. through the use of a stable Long Term Forward Rate). These are two key considerations when setting yield curves for the purpose of discounting insurance liabilities.

2016 ICS Consultation Feedback

- Largely respondents supported the three segment approach, with suggestions for alignment with IFRS 17

2.2. Methodology for ICS Version 2.0

The three segment approach as described in Section 2.1 will be kept.

Each yield curve is split into three segments:

- Segment 1: Yields for liquid tenors are based on market information. The market information is taken from liquid instruments that are either interest rate swaps or government bonds (see Section 3). A method (see Section 7) fits the yield curve exactly at the liquid tenors and interpolates between them. The Last Observed Term (LOT) is the last tenor with an exact fit to liquid market data (currently limited to 30 years).
- Segment 2: Yields after the LOT are extrapolated until the convergence point (currently at a tenor of 60 years for all currencies).
- Segment 3: From the convergence point of the LTFR onwards, the corresponding forward intensity of the yield curves converged within the respective constraints to the LTFR intensity and stays within the limits afterwards.

Refinements to the methodology:

The base yield curves for each currency are derived using an inter-/extrapolation method (see Section 7). Therefore, each yield curve is split into three segments:

- Segment 1: The last observed term (LOT) is the last tenor with an exact fit to liquid market data and can be any tenor for which the liquidity criteria are still met (see Section 3). In practice, this means the elimination of the 30 year cap for the length of Segment 1 of the yield curve.
- Segment 2: Yields after the LOT are extrapolated until the convergence point of the LTFR at a tenor defined in accordance with Section 6. In practice, this means the elimination of the mandatory convergence at 60 years, for all currencies.
- Segment 3: No changes to the approach.

3. Choice of instrument for Segment 1

3.1. Previous Approach

The first segment of the curve is based on market information, using either Government bond or swap data.

The choice of the instrument was initially grounded on liquidity considerations, as well as the perceived representativeness of the investments held by the insurance companies operating in that jurisdiction.

The chosen instrument underlying Segment 1 was subsequently allowed to be changed, for the purpose of Field Testing, based on supervisory judgment.

Table 1 - Instruments used for Segment 1 in 2017 Field Testing

Code	Currency name	SWP/GVT?	Ticker
AUD	Australian Dollar	GVT	I00101Y Index
BRL	Brazilian Real	GVT	I39301Y INDEX
CAD	Canadian Dollar	SWP	CDSW1 CMPN Curncy
CHF	Swiss Franc	SWP	SFSW1 CMPL Curncy
CLP	Chilean Peso	SWP	CHSWP1 CMPN Curncy
CNY	Yuan Renminbi	GVT	http://yield.chinabond.com.cn/cbweb-mn/yield_main
COP	Colombian Peso	SWP	CLSWU1 CMPN Curncy
CZK	Czech Koruna	SWP	CKSW1 CMPL Curncy
DKK	Danish Krone	SWP	DKSW1 CMPL Curncy
EUR	Euro	SWP	EUSA1 CMPL Curncy
GBP	Pound Sterling	SWP	BPSW1 CMPL Curncy
HKD	Hong Kong Dollar	SWP	HDSW1 CMPT Curncy
HUF	Forint	GVT	I16501Y INDEX
IDR	Rupiah	SWP	IHSWN1 CMPT Curncy
ILS	New Israeli Sheqel	SWP	ISSW1 CMPL Curncy
INR	Indian Rupee	SWP	IRSWM2 CMPT Curncy
JPY	Yen	GVT	I01801Y Index
KRW	Won	GVT	GVSK1yr Index
MXN	Mexican Peso	GVT	I25101Y INDEX
MYR	Malaysian Ringgit	GVT	I19601Y Index
NOK	Norwegian Krone	SWP	NKSW1 CMPL Curncy
NZD	New Zealand Dollar	SWP	NDSW1 CMPT Curncy
PEN	Sol	SWP	PSSWN1 CMPN Curncy
PHP	Philippine Peso	SWP	PPSWO1 CMPT Curncy
PLN	Zloty	GVT	I17701Y INDEX
RON	Romanian Leu	GVT	C4891Y CMPL Index
RUB	Russian Ruble	SWP	RRSWM1 CMPL Curncy
SAR	Saudi Riyal	SWP	SRSW1 CMPL Curncy
SEK	Swedish Krona	SWP	SKSW1 CMPL Curncy
SGD	Singapore Dollar	GVT	I10701Y Index
THB	Baht	GVT	F12201Y index
TRY	Turkish Lira	GVT	BV020965 INDEX
TWD	New Taiwan Dollar	GVT	F12601Y Index
USD	US Dollar	GVT	I11101Y Index
ZAR	Rand	GVT	F26201Y Index

2016 ICS Consultation Feedback

- IAIS requested specific feedback on the appropriateness of the instrument chosen (swaps or government bonds), rather than on the methodology to perform the choice.

3.2.   Methodology for Version ICS 2.0

The base yield curves published by the IAIS should be derived from financial instruments that are traded in deep, liquid and transparent (DLT) financial markets. A DLT assessment should be carried out at regular intervals (e.g. annually, every 2-3 years) in order to identify the financial instruments and maturities for which a DLT market exists. The DLT assessment determines, in particular, whether swaps or government bonds are the relevant financial instruments for the risk-free interest rates and what the LOT is when the extrapolation of the base yield curve starts.

The DLT assessment needs to be performed for each currency for which the IAIS provides base yield curves.

There are several factors influencing the liquidity (and depth) of financial markets. Further, the influence of these factors varies across markets (e.g. according to their practices, conventions and operational rules) and also varies over time within the same market (e.g. according to changes in the environment). Finding a generalized way to measure the level of these factors is challenging and the subject of continuing research. It is generally accepted that no single metric can be conclusive in assessing the DLT nature of a financial instrument.

Components of the DLT assessment

The DLT assessment should consist of the components set out in the following table:

Component	Purpose
DLT assessment of the swap market	Decision on relevant financial instrument (swaps/government bonds) and DLT maturities
DLT assessment of the government bond market	

The following sections set out details on the possible criteria to be used for each component of the assessment.

DLT assessment of the swap market

Depth and liquidity

The assessment of depth and liquidity of the swap market should be carried out on the basis of swap trade data, in particular the number and notional amount of trades. As each of the depth, liquidity and transparency criteria lacks a globally accepted clear definition that is of practical use, a non-exhaustive list of other potential criteria is provided, which could be used to help supervisors for their view. In order to ensure an assessment that is consistent across

currencies it should be made in accordance with criteria that are as objective and clearly specified as possible.

The starting point for the assessment could be the following thresholds for depth and liquidity:

- The one-year average daily notional amount traded is at least EUR 50 000 000,

- The one-year average daily number of trades is at least ten.

Only single-currency fixed-to-floating swaps should be considered for assessing the criteria. The assessment should be made separately for each currency and maturity. Where possible the thresholds should be assessed on the basis of data that cover the period of one year. Furthermore, the thresholds may need to be adapted in light of the specific circumstances of individual markets (e.g. less liquid currencies).

The following are additional criteria that may be used:

a. Bid-ask spread: the price difference between the highest price a buyer would pay and the lowest price for which a seller would sell

b. Swap rate volatility: the behaviour of the available interest rates for each maturity over a certain number of days

c. Trader quotes/dealer surveys (incl. dispersion of answers);

d. Quote counts (1): number of dealer quotes within a window of a few days;

e. Quote counts (2): number of dealers quoting

f. Number of pricing sources

g. Assessment of large trades and movement of prices (depth)

h. Matching analysis (assessing the ability of IAIGs to match with bonds the cash flows which are discounted with non-extrapolated interest rates in the calculation of the current estimate)

In order to ensure the stability of the DLT maturities, clear criteria should be set to establish the conditions under which the status should be changed. For example, a market assessed liquid in the past should only be considered to have become illiquid if it is at least 20% below one of the two thresholds set out above. A market assessed illiquid in the past should only be considered to have become liquid if it is at least 20% above the two thresholds set out above.

This corridor around the thresholds prevents, for example, that a market with continuously more than ten daily trades and a development of average daily notional amounts of EUR 49 000 000, EUR 51 000 000 and EUR 49 000 000 in consecutive assessments is considered to change its liquidity status every time.

Trade numbers and notional amounts should be scaled up to allow for trade not covered by the data. The scaling factors can be derived from the triannual OTC derivative statistics of the Bank for International Settlements.

DLT assessment of the government bond market

Depth and liquidity

Given the specificity of government bond markets, a different approach needs to be followed to assess the DLT nature of government bond rates.

The assessment could still start from the analysis of trade volume and trade frequency of government bonds, for all currencies. Decisions on depth and liquidity of the government bond market could be based primarily on these criteria. Where possible, the assessment should be based on data that cover the period of one year.

Where trade volume and frequency data are not available or their analysis is not conclusive, other criteria could be assessed, including where possible, bid-ask spreads, the rate volatility, zero-trading days, the number of pricing sources and the number of quotes.

Conclusions from the assessments

The DLT assessment of the relevant instruments will inform about the choice of the instrument on which the base yield curve should be built upon, as well as the DLT maturities that could be used in the derivation of the base yield curve.

To maximize the use of market information, it could be justified to choose the instrument with the longer DLT segment or the instrument with a higher overall degree of liquidity.

Given the specificities of the financial markets of each currency, the jurisdictional supervisor will be in the best position to finally decide which instrument should be taken as a basis, keeping in mind the key objectives of the MAV approach. An appropriate justification for this choice should be presented, covering also the outcome of the DLT analysis.

4. Last Observed Term

4.1. Previous Approach

Segment 1 of the base yield curve ends at the point at which the selected market instruments are no longer available, or are not deemed to be traded in deep, liquid and transparent markets.

The initial LOT have been chosen by the IAIS based on publicly available supervisory information and have been adjusted over time based on judgment from the jurisdictional supervisory authorities.

For the time being, no currency has a LOT beyond 30 years. This is mainly to ensure a smooth extrapolation to the point of convergence with the Long Term Forward Rate (LTFR), which is fixed at a tenor of 60 years for all currencies.

Table 2 - Last Observed Term for all Yield Curves in 2017 Field Testing

Code	Currency name	LOT

AUD	Australian Dollar	30
BRL	Brazilian Real	10
CAD	Canadian Dollar	20
CHF	Swiss Franc	20
CLP	Chilean Peso	10
CNY	Yuan Renminbi	10
COP	Colombian Peso	10
CZK	Czech Koruna	15
DKK	Danish Krone	20
EUR	Euro	20
GBP	Pound Sterling	30
HKD	Hong Kong Dollar	15
HUF	Forint	15
IDR	Rupiah	10
ILS	New Israeli Sheqel	20
INR	Indian Rupee	10
JPY	Yen	30
KRW	Won	20
MXN	Mexican Peso	20
MYR	Malaysian Ringgit	15
NOK	Norwegian Krone	10
NZD	New Zealand Dollar	20
PEN	Sol	10
PHP	Philippine Peso	10
PLN	Zloty	10
RON	Romanian Leu	10
RUB	Russian Ruble	10
SAR	Saudi Riyal	15

SEK	Swedish Krona	10
SGD	Singapore Dollar	20
THB	Baht	10
TRY	Turkish Lira	10
TWD	New Taiwan Dollar	10
USD	US Dollar	30
ZAR	Rand	30

2016 ICS Consultation Feedback

- IAIS requested specific feedback on the appropriateness of the LOT chosen, rather than on the methodology to define it.

4.2. Methodology for ICS Version 2.0

As a central element of the MAV valuation, the base yield curve should, to the extent possible, contribute to the objective of avoiding artificial volatility of insurance liabilities and capital resources and provide an incentive for good risk management.

Following the DLT assessment and the choice of the instrument underlying segment one of the yield curve, the LOT is set as the longest maturity which is deemed to fulfil the DLT criteria, for the chosen instrument.

5. Credit Risk Adjustment (CRA)

5.1. Previous Approach

In the current IAIS methodology, a Credit Risk Adjustment (CRA) is applied only to base yield curves based on swap rates.

The CRA is applied as a parallel downward shift of 10bps to the market rates observed for maturities up to the last observed term. The 10 bps have been defined using expert judgement and are applied to all relevant currencies.

Regarding swaps, the CRA is applied to the observed par swap rates before deriving zero coupon rates. In the case of base yield curves based on government bond rates, no CRA is applied.

The exclusion of the CRA for base yield curves based on government bond rates follows the assumption that these instruments are risk-free.

5.2. Methodology for Version ICS 2.0

The CRA relates to the fact that the reference rates used as floating legs in the swap agreements carry counterparty credit risk, since they originate from unsecured interbank market transactions. For example, the floating leg of Euro area swaps is based on Euribor rates. Given that the floating leg reflects counterparty credit risk, the fixed leg will also carry credit risk, since in an efficient market the fixed leg will be based on expectations of future realisations of the floating rate over the duration of the swap arrangement.

For the sake of simplicity, the CRA has been set for Field Testing at 10 basis points for all currencies. Although that value may differ from currency to currency, when calculated based on market information (comparing the underlying rates with OIS rates), such calculation would likely face significant limitations in terms of data availability.

For that reason, the CRA will remain at 10 basis points for all currencies.

6. Length of Segment 2

6.1. Previous Approach

The convergence point, i.e. the maturity at which the forward rates of the base yield curves converge to the LTFR (or the end of Segment 2), has been set at 60 years for all currencies, irrespectively of the tenor which is defined as the end of Segment 1.

This was done primarily for two reasons: on one hand, to simplify the approach and, on the other, due to the link with the methodology underlying the LTFR calculation, which looks at long-term macroeconomic equilibrium, and is assumed to be reached only in the distant future, e.g. in 60 years.

Under this methodology, the length of Segment 2 of the base yield curve is therefore variable. It is given by the distance between the LOT and the 60 year maturity, for each currency (in practice, it ranges between 30 and 50 years).

2016 Public Consultation Feedback

- Some support for the current approach but also many responses expressing criticism/alternative proposals

- Some supported a fixed length

- A small number suggested a proportional approach, though sometimes with opposite interpretations

- Some supported the combined approach (minimum length + minimum convergence tenor)

6.2. Methodology for ICS Version 2.0

The length of Segment 2 determines the speed of convergence to the LTFR. Therefore, a shorter Segment 2 would result in base yield curves that are dominated by the LTFR in the mid and long end. Especially for yield curves with a longer LOT and lower observable rates,

this could result in steep and wavy behaviour of the curves that may not be aligned with observed but unused market rates in the long end.

An appropriate balance should be achieved between, on the one side, the need to ensure a sufficiently long convergence period in all cases which avoids a steep convergence to the LTFR (for currencies with a long Segment 1) and, on the other side, the need to ensure that the LTFR is not reached very early in the base yield curve (as the LTFR is expected to reflect long term stable expectations), in particular for currencies with a very short Segment 1).

To achieve these conflicting objectives, a combined approach could be used. In practice, a minimum convergence point (60 years) would be considered together with a consistent convergence time (30 years). This would simultaneously ensure that no convergence occurs before the defined minimum convergence point and that in no circumstance can the convergence period be lower than 30 years (without the need to artificially constrain the LOT which is embedded in the current IAIS approach).

Mathematically, the length of segment 2 is:

$$\text{Length}_{\text{Segment 2}} = \max(minimum\ convergence\ point - LOT, consistent\ convergence\ time).$$

7.   Extrapolation/Interpolation method

7.1.   Previous Approach
In past field testing exercises, extrapolation of the base yield curves produced by the IAIS has been performed using the Smith-Wilson methodology.

7.2.   Methodology for ICS Version 2.0

Both the interpolation between Segment 1 maturities and the extrapolation beyond the LOT will continue to be based on the Smith-Wilson methodology.

This is because of the Smith-Wilson methodology's favourable properties of incorporating the LTFR as well as exact replications of the observed liquid market data.

The control input parameters for the interpolation and extrapolation are the last point of liquidity, long-term forward rate (LTFR), the convergence point and the convergence tolerance.

In order to apply the Smith-Wilson method, a cash-flow matrix is derived from the observed market interest rate data. The Smith-Wilson method takes care that the present value function of the derived yield curve exactly agrees with the empirical data for the observable maturities.

If the reference instruments are swap rates, the market interest rates to be used as inputs should be the swap par rates after deduction of the credit and currency risk adjustments. If the reference instruments are zero coupon government bonds, the market interest rates to be used as inputs should be the zero coupon rates.

The parameter alpha that controls the convergence speed is set at the lowest value that produces a yield curve reaching the convergence tolerance of the LTFR by the convergence point. A lower bound for alpha could be set at 0.05.

Further technical details about the Smith-Wilson methodology are provided in Annex 1.

8. Convergence tolerance

8.1. Previous Approach

So far, the IAIS has determined its yield curves by setting the convergence tolerance parameter at 1 basis point at maturity 60 (the convergence point). This means that, at this tenor, the forward rate produced by the model is less than 1 basis point higher or lower than the target LTFR.

8.2. Methodology for ICS Version 2.0

For simplicity, the convergence tolerance will remain at 1 basis point, and will be achieved at the tenor which marks the end of Segment 2 (see Section 6).

9. LTFR components

9.1. Previous Approach

The LTFR is assumed to represent the nominal rate expected to be earned when economies reach their long-term macroeconomic equilibrium.

In past Field Testing exercises, the LTFR for each currency was based on long-term expectations of economic growth and long-term expectations of inflation for the relevant economies. The long-term expectations of economic growth are derived from an OECD study[1]. In that study, OECD economies are forecasted to reach an annual growth of 1.5% per annum in 2060 and non-OECD countries are forecasted to reach an annual growth of 2.75% per annum in 2060. When combined with inflation targets of the central banks, a LTFR figure can then be derived. For example, for Australia the inflation target is 2.5% and the growth expectation is 1.5% which leads to a 4% long-term forward rate. For Brazil, it is a non-OECD country with a 2.75% growth expectation and long-term inflation expectation of 4.5% leading to an LTFR of 7.3% (rounded up).

Table 3 - Long-Term Forward Rates per Base Yield Curve

Code	Currency name	LTFR
AUD	Australian Dollar	4.00%
BRL	Brazilian Real	7.25%
CAD	Canadian Dollar	3.50%

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[1] https://www.oecd.org/eco/outlook/2060%20policy%20paper%20FINAL.pdf

CHF	Swiss Franc	3.50%
CLP	Chilean Peso	4.50%
CNY	Yuan Renminbi	6.75%
COP	Colombian Peso	4.50%
CZK	Czech Koruna	3.50%
DKK	Danish Krone	3.50%
EUR	Euro	3.50%
GBP	Pound Sterling	3.50%
HKD	Hong Kong Dollar	3.50%
HUF	Forint	4.50%
IDR	Rupiah	7.25%
ILS	New Israeli Sheqel	3.50%
INR	Indian Rupee	6.75%
JPY	Yen	3.50%
KRW	Won	4.50%
MXN	Mexican Peso	4.50%
MYR	Malaysian Ringgit	7.25%
NOK	Norwegian Krone	4.00%
NZD	New Zealand Dollar	3.50%
PEN	Sol	4.50%
PHP	Philippine Peso	7.25%
PLN	Zloty	4.00%
RON	Romanian Leu	3.50%
RUB	Russian Ruble	6.75%
SAR	Saudi Riyal	4.75%
SEK	Swedish Krona	3.50%
SGD	Singapore Dollar	3.50%
THB	Baht	7.25%

TRY	Turkish Lira	6.50%
TWD	New Taiwan Dollar	6.75%
USD	US Dollar	3.50%
ZAR	Rand	7.25%

> 2016 ICS Consultation Feedback
>
> - Substantial support for the macroeconomic approach, with some challenging the variables used, the reliance on prospective analysis and the dependence of the OECD study

9.2. Methodology for ICS Version 2.0

The LTFR should continue to be calculated on the basis of a macroeconomic approach, as this enables the production of base yield curves that are stable at the long end. Such an approach reduces volatility in the balance sheet of insurers writing long term business, driven by market fluctuations observed in shorter term maturities. In this way, the LTFR methodology would support some of the objectives defined for the ICS.

Refinements to the methodology:

The long-term expectation of economic growth component of the LTFR will be replaced with expected real interest rates, i.e.

$$LTFR = Expected\ Inflation\ Rate + Expected\ Real\ Interest\ Rate$$

The current approach relies on the assumption that future economic growth is approximately equal to future real interest rates and projections of it could be therefore used as a proxy. However, recent academic research suggests that there may be only a weak correlation between these two quantities.[2]

In addition, the methodology relies on a model of an OECD study which may not be fully reproducible by the IAIS. This would significantly constrain the ability of the IAIS to monitor, predict changes and update the LTFR.

Apart from enhanced transparency and reproducibility, the revised methodology is better aligned with economic theory by taking real interest rates directly.

The data source and the calculation of the components is described in the following two subsections.

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[2] See e.g. Hansen, Bruce E. and Ananth Seshadri, 2013, Uncovering the Relationship between Real Interest Rates and Economic Growth, Ann Arbor MI: University of Michigan Retirement Research Center (MRRC) Working Paper, WP 2013 - 303.

Calculation of the expected real interest rate

The expected real interest rate is the long term simple arithmetic mean of annual real rates (e.g. starting in 1960 to the year before the recalculation of the LTFRs) according to the following formula:

$$R = \frac{1}{n}\sum_{i=1}^{n} r_{1960+i}$$

where:

- R is the expected real rate,
- n is the number of years since end of 1960,
- r_i is the annual real rate for the year 1960+i,

For each of the years the annual real rate is derived as the simple arithmetic mean of the annual real rates observed for that currency. In the case of currency unions, a proxy will need to be developed.

For each of those years and each country, the annual real rate is calculated as follows:

$$Real\ rate \ = \frac{short\ term\ nominal\ rate \ - \ inflation\ rate}{1 \ + \ inflation\ rate}$$

The short-term nominal rates and inflation rates are taken from the annual macro-economic database of the European Commission (AMECO database)[3] and from the Main Economic Indicators database of the OECD.[4]

The expected real rate should be rounded to avoid spurious accuracy (e.g. to the nearest full five basis points).

Calculation of the expected inflation rate

For currencies where the central bank has announced an inflation target, the expected inflation is based on that inflation target. For simplicity, a limited number of inflation buckets could be defined according to the following rules:

- The expected inflation rate is:
    - 1%, where the inflation target is lower than or equal to 1%,
    - 2%, where the inflation target is higher than 1% and lower than 3%,
    - 3%, where the inflation target is higher or equal to 3% and lower than 4%,

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[3] Short-term nominal rates used for deriving the expected real rate can be found in the annual macro-economic database of the European Commission's Directorate General for Economic and Financial Affairs, "AMECO". On AMECO online, select 13-Monetary variables, select Interest Rates and then tick the box Short-term nominal (ISN). (http://ec.europa.eu/economy_finance/ameco/user/serie/ResultSerie.cfm)

[4] Inflation rates used for deriving the expected real rate can be found on the website of the Organisation for Economic Co-operation and Development (OECD): go to the OECD Main Economic Indicators (MEI) and select consumer price indices. When accessing the database, choose consumer prices – all items for the subject, percentage change on the same period of the previous year for the measure and percentage for the unit. (http://stats.oecd.org/Index.aspx?DataSetCode=MEI_PRICES). OECD data used in this document were accessed in March 2016.

    o   4%, where the inflation target is 4% or higher.

- Where a central bank is not targeting a specific inflation figure but tries to keep the inflation in a specified corridor, the midpoint of that corridor would be relevant for the allocation to the four inflation rate buckets.

For currencies where the central bank has not announced an inflation target, the expected inflation rate could be set at 2% by default. However, where past inflation experience (e.g. looking at 10 years of data) and projection of inflation both clearly indicate that the inflation of a currency is expected, in the long-term, to be materially higher or lower than 2% (e.g. by more than 1 percentage point), the expected inflation rate should be chosen in accordance with those indications (rounded downwards to full percentage points, for simplicity).

Practical implementation

In practical terms, it is not possible to implement the methodology specified in the previous sub-sections for all of the 35 currencies for which the IAIS publishes base yield curves. This is due to a lack of data. For this reason, a simplified approach was adopted, for the calculation of the real interest rate component. Two figures were derived, for Developed Markets and for Emerging Markets, according to the following criteria:

- Developed Markets: long term simple arithmetic mean of annual real rates with the minimum period of historical data of 45 years. The result was 1.8%.
- Emerging Markets: long term simple arithmetic mean of annual real rates with the minimum period of historical data of 20 years. The result was 3%.

Annex 2 includes further details on the data used to derive the figures provided.

10. LTFR updates

10.1. Previous Approach

The issue of LTFR updates has not been considered so far by the IAIS.

By design, under the current approach, the LTFR is defined to reflect long term economic conditions of each jurisdiction. It is therefore expected to remain relatively stable over time.

However, even long term expectations are expected to change over time, as economic fundamentals evolve. This means that, although stability is a key feature of the LTFR concept, it should not be interpreted that such figure should remain unchanged for the foreseeable future.

Therefore, the IAIS should agree on a methodology which establishes the frequency and magnitude (i.e. by how much can the LTFR change each time it is updated) of potential LTFR updates.

2016 ICS Consultation Feedback

10.2. Methodology for ICS Version 2.0

On this issue, it is necessary to bring together two conflicting objectives. On the one side, the stable nature of the LTFR concept and, on the other, the need to derive a figure that still reflects long term expectations in a credible manner. Against this background, changes to the LTFR, as a result of the application of the methodology described in Section 9, will be allowed but such changes will be limited to a relatively small figure in order to prevent disruptive dislocations of the base yield curve. To better reflect the stability objective, the LTFR will be kept unchanged if the outcome of the recalculation would not materially differ from the figure currently being used. This would avoid spurious changes of the LTFR (e.g. by 1 or 2 basis points) on a very frequent basis.

For such an approach to work, a limit of 15 bps is set. Under this approach, for each currency, the change of the LTFR is limited in such a way that it increases or decreases by 15 bps or remains unchanged in accordance with the following rule:

$$LTFR_t^L = \begin{cases} LTFR_{t-1}^L + 15\ bps & if\ \ LTFR_t \geq LTFR_{t-1}^L + 15\ bps \\ LTFR_{t-1}^L - 15\ bps & if\ \ LTFR_t \leq LTFR_{t-1}^L - 15\ bps \\ \quad LTFR_{t-1}^L & \qquad otherwise \end{cases}$$

where:

- $LTFR_t^L$ denotes the LTFR of year t, after limitation of the annual change,
- $LTFR_{t-1}^L$ denotes the LTFR of year t+1 , after limitation of the annual change,
- $LTFR_t$ denotes the LTFR of year t, before limitation of the annual change.

Annex 1 - Description of the Smith-Wilson inter-/extrapolation

Wilson function

The Wilson function W(u,v) is specified as:

$$W(u,v) = e^{-\omega(u+v)}H(u,v)$$

where H(u,v) is the Wilson heart function:

$$H(u,v) = \alpha\min(u,v) - e^{-\alpha\max(u,v)}\cdot\sinh(\alpha\min(u,v)).$$

The parameter ω denotes the long term forward intensity and is the natural logarithm of the annualized LTFR. The α parameter is a convergence parameter and its calibration will be described in section xxx.

The H-function and its derivative are continuous at u=v:

$$H(u,v) = \alpha\min(u,v) - \exp(-\alpha\max(u,v))\cdot\sinh(\alpha\min(u,v))$$

Differentiation with respect to *v* gives:

$$G(u,v) := \frac{dH(u,v)}{dv} = \begin{cases} \alpha e^{-\alpha v}\sinh(\alpha u) & u \le v \\ \alpha - \alpha e^{-\alpha u}\cosh(\alpha v) & u > v \end{cases}$$

Matrices and vectors

Matrices and vectors will be boldface. Transposition is indicated by a prime and $\circ$ denotes element-wise multiplication of conformable matrices. 1 and 0 will denote column vectors with all components equal to 1 and 0 respectively, and of appropriate order.

A vector u for the m observed durations to maturity is introduced, as well as an m×n matrix C that for the cash-flows of the n financial instruments:

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ M \\ u_m \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & L & c_{1n} \\ c_{21} & c_{22} & L & c_{2n} \\ M & M & & M \\ c_{m1} & c_{m2} & L & c_{mn} \end{bmatrix} \qquad c_{ij} \ge 0$$

$$\mathbf{d} = \exp[-\omega\mathbf{u}] = \begin{bmatrix} e^{-\omega u_1} \\ e^{-\omega u_2} \\ M \\ e^{-\omega u_m} \end{bmatrix} \qquad p[\mathbf{u}] = \begin{bmatrix} p(u_1) \\ p(u_2) \\ M \\ p(u_m) \end{bmatrix} \qquad \sinh[\alpha\mathbf{u}] = \frac{1}{2}\begin{bmatrix} e^{\alpha u_1} - e^{-\alpha u_1} \\ e^{\alpha u_2} - e^{-\alpha u_2} \\ M \\ e^{\alpha u_m} - e^{-\alpha u_m} \end{bmatrix}$$

An auxiliary matrix $\mathbf{Q} = \mathbf{d}_\Delta \mathbf{C}$ will be needed where the subscript Δ denotes transforming a column vector into a diagonal matrix such that $\mathbf{d}_\Delta \mathbf{1} = \mathbf{d}$. Furthermore there are the following three column vectors with n components:

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \mathsf{M} \\ b_n \end{bmatrix} \qquad \mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \\ \mathsf{M} \\ p_n \end{bmatrix} \qquad \mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ \mathsf{M} \\ q_n \end{bmatrix} = \mathbf{Q}'\mathbf{1} = \mathbf{C}'\mathbf{d}$$

Here b is an auxiliary matrix and p contains the n observed market prices for the n financial instruments that will be compared with the m components of the present values in p[u].

Zero-rows in C can be deleted from the tableau without loss of generality. In case of non-deletion this will imply zero components in the output vector Qb at the appropriate places.

Wilson matrix and H-matrix

On that basis of the definitions made above, the following can be displayed:

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \mathsf{M} \\ v_k \end{bmatrix} \qquad \mathbf{W}(\mathbf{v},\mathbf{u}) = \begin{bmatrix} W(v_1,u_1) & W(v_1,u_2) & \mathsf{L} & W(v_1,u_m) \\ W(v_2,u_1) & W(v_2,u_2) & \mathsf{L} & W(v_2,u_m) \\ \mathsf{M} & \mathsf{M} & & \mathsf{M} \\ W(v_k,u_1) & W(v_k,u_2) & \mathsf{L} & W(v_k,u_m) \end{bmatrix} = \mathbf{W}'(\mathbf{u},\mathbf{v})$$

$$\mathbf{W}(v,\mathbf{u}) = \begin{bmatrix} W(v,u_1) & W(v,u_2) & \mathsf{L} & W(v,u_m) \end{bmatrix} = \mathbf{W}'(\mathbf{u},v)$$

$$\mathbf{W}(\mathbf{u},\mathbf{u}) = \mathbf{W} = \mathbf{d}_\Delta \mathbf{H} \mathbf{d}_\Delta = \mathbf{H} \mathsf{o} \mathbf{d}\mathbf{d}'$$

The symmetric matrices W and H will be positive definite as long as u contains distinct positive components.

Smith-Wilson present value function

This function, also known as discount pricing function, can be displayed as:

$$p(v) = e^{-\omega v} + \mathbf{W}(v,\mathbf{u})\mathbf{Cb} = e^{-\omega v} + e^{-\omega v}\mathbf{H}(v,\mathbf{u})\mathbf{Qb}$$

where the values for u correspond to the observed durations to maturity of the financial instruments and v is the duration to maturity of the present value function.

A set of equations can be formed by having v the values of u:

$$p[\mathbf{u}] = \exp[-\omega\mathbf{u}] + \mathbf{WCb} = \mathbf{d} + \mathbf{WCb} = \mathbf{d} + \mathbf{d}_\Delta \mathbf{H} \mathbf{d}_\Delta \mathbf{Cb} = \mathbf{d} + \mathbf{d}_\Delta \mathbf{HQb}$$

Pre-multiplication with the transpose of C gives n linear equations in b:

$$\mathbf{C}'p[\mathbf{u}] = \mathbf{C}'\mathbf{d} + \mathbf{C}'\mathbf{WCb} = \mathbf{q} + \mathbf{Q}'\mathbf{HQb}$$

p is the market observable counterpart of $\mathbf{C}'p[\mathbf{u}]$

$$\mathbf{p} = \mathbf{q} + \mathbf{Q}'\mathbf{HQb}$$

From this follows the solution for b:

$$\mathbf{b} = (\mathbf{Q}'\mathbf{HQ})^{-1}(\mathbf{p} - \mathbf{q})$$

This solution depends on ω through Q and q as well as on α through H. The value for α will be determined through convergence requirements. The input parameters for different financial instruments are given in Table 4.

Table 4 - Parameters used for the inter-/extrapolations of different financial instruments

Instruments	Market prices p	Cash payment dates u	Cash-flow matrix C
Zero coupon bonds	Market prices of the n input instruments, given as the percent amount of the notional amount The market prices of the zero coupon input bonds translate at once into spot rates for input maturities	The cash payment dates are the maturity dates of the n zero coupon input bonds (i.e. m=n)	An n×n matrix with entries: - cij =1 for i=j, - cij =0 else. C is the identity matrix.
Coupon bonds	Market prices of the n coupon input bonds, given as the percent amount of the notional amount of the bond.	The cash payment dates are, in addition to the maturity dates of the input bonds all coupon dates.	An m×n matrix with entries: - cij =rc(i)/s, i<t(j) - ct(j),j =1+rc(i)/s, - cij=0, i>t(j), where rc(i) is the coupon rate of bond i, s is the settlement frequency

			and t(j) the maturity of bond j.
Par swap rates	The market prices of the n par swap input instruments are taken as unit (i.e. 1). To receive the swap rate, a floating rate has to be earned, that can be swapped against the fixed rate. To earn the variable rate a notional amount has to be invested. At maturity, the notional amount is de-invested.	The cash payment dates are, in addition to the maturity dates of the swap agreements all swap rate payment dates.	An m×n matrix with entries: - cij =rc(i)/s, i<t(j) - ct(j),j =1+rc(i)/s, - cij =0, i>t(j), where rc(i) is the swap rate of agreement i, and s is the settlement frequency and t(j) the maturity of arrangement j.

Smith-Wilson yield and forward intensity function

The yield intensity function follows as:

$$y(v) = \frac{-\log p(v)}{v} = \omega - \frac{\log(1 + \mathbf{H}(v,\mathbf{u})\mathbf{Qb})}{v}$$

The forward intensity function follows as

$$f(v) = \frac{-d \log p(v)}{dv} = \omega - \frac{d \log(1 + \mathbf{H}(v,\mathbf{u})\mathbf{Qb})}{dv} = \omega - \frac{\mathbf{G}(v,\mathbf{u})\mathbf{Qb}}{1 + \mathbf{H}(v,\mathbf{u})\mathbf{Qb}}.$$

Analysis of convergence to the long-term forward intensity

For $v \geq U = \max(\mathbf{u})$ we have:

$$\mathbf{H}(\mathbf{u},v) = \alpha\mathbf{u} - e^{-\alpha v}\sinh[\alpha\mathbf{u}] \qquad \mathbf{G}(\mathbf{u},v) = \alpha e^{-\alpha v}\sinh[\alpha\mathbf{u}]$$

Now, the upper end of the forward intensity function reduces to:

$$f(v) = \omega + \frac{\alpha}{1 - \kappa \cdot e^{\alpha v}} \qquad v \geq U$$

where κ is a quasi-constant that depends on α (and ω) but not on v:

$$\kappa = \frac{1 + \alpha \mathbf{u}'\mathbf{Qb}}{\sinh[\alpha \mathbf{u}']\mathbf{Qb}}.$$

If α is such that κ=0, then $f(v) = \omega + \alpha$, irrespective of the value of v and the ultimate forward intensity $f(\infty)$ will not approach ω.

The value of α is determined by requirements on the convergence speed and will automatically be chosen in such a way that $\kappa \neq 0$.

Adopting a length of segment 2 $S = \max(40, 60 - U)$ implies a point of convergence T as follows:

$$T = U + S = \max(U + 40, 60)$$

The convergence gap at the point of convergence T can be analysed as a function of α:

$$g(\alpha) = |f(T) - \omega| = \frac{\alpha}{|1 - \kappa e^{\alpha T}|}$$

and the problem of determining α can be formulated as a nonlinear minimization problem:

Minimize α with respect to α subject to the two inequality conditions:

(1) $\alpha \geq a$ with the lower bound a =0.05

(2) $g(\alpha) \leq \tau$.

A heuristic solution strategy is the following:

if $\alpha = a$ implies $g(\alpha) \leq \tau$ then $\alpha = a$ is optimal

else search for $\alpha > a$ such that $g(\alpha) = \tau$

Annex 2 - Data used in the derivation of the Real Interest Rate Component of the LTFR

Developed Markets

Currency	Years of Data Used
AUD	49
CAD	56
EUR	56 for 5 EUR countries (BE, DE, FR, NL, IT)
GBP	56
JPY	47
USD	56

Emerging Markets

Currency	Years of Data Used
CNY	17
COP	31
MXN	20
RUB	20
TRY	20
ZAR	36